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Mathematical Model of Equilibrium Prices and Quantities for Four Interdependent Goods in a Commodity Market

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Abstract

This Isolated market is a market where both quantities demanded and quantities supplied are functions of one commodity alone. This type of market hardly occurs in our present-day situation. Before now, coca cola is almost the sole producer of soft drinks in Nigeria, but today there a lot of companies producing soft drinks. Every commodity now has two or more substitutes thereby making the adjustment of price easy. Many models developed in the time past were mainly used to determine the equilibrium of quantities supplied and quantities demanded for one commodity. Since the market system has changed from one commodity to two or more commodities, the model has to be changed also. So, in this research work, we developed a model Cramer's rule to determine the state of equilibrium for four interdepended goods in a commodity market. The Cramer's rule is matrix form that uses the determinant method in solving the system of linear equations. The equilibrium prices for each of the commodities were derived. Also derived, were the equilibrium quantities. A numerical example to test the model was also given. At the various price derived from the model, equilibrium for the four commodities were obtained. From the result, the quantities supplied and demanded at the market existing are as follows: Apple is 74, Grape is 146, Coffee is 99 and that of Banana is 59. Since the market is being control by price, consumers with low income can easily move from one good to another to satisfy their immediate needs based on their income. In this situation, it is no longer the quality that matters but the quantity. Having a good knowledge of the equilibrium quantity at existing market price will help producers to produce exactly the quantity needed by consumers thereby avoiding wastage of goods.

Keywords*:* Commodity Market, Interdependent goods, Equilibrium Price, Determinant of a Matrix, Cramer's Rule.

Introduction:

A commodity market is a marketplace for buying, selling and trading raw materials or primary products. There are currently about 50 major commodity markets worldwide that facilitate trade in approximately 100 primary commodities (Adam, 2021).

Commodities are often divided into two major groups: hard and soft commodities. Hard commodities include natural resources that must be mined or extracted such as gold, rubber and oil while soft commodities are agricultural products or livestock such as corn, wheat, coffee, sugar, soybeans and pork (Moosa, 2000).

Commodities markets allow producers and consumers of commodity products to gain access to them in a centralized and liquid marketplace. These market actors can also use commodities derivatives to hedge future. Speculators, investors and arbitrageurs also play an active role in these markets.

Interdependent Goods

Interdependent goods are those goods or services whose demand or supply are influenced by the demand or supply of other goods. Interdepended goods are very significance in a commodity market in the sense that it is use to understand how changes in one good’s price affects the demand and supply of another good. It also helps to identify products that are used together, like peanut butter and jelly, to create market opportunities. Interdepended goods inform pricing decisions, such as bundling or discounting

Types of Commodity Markets

Generally, commodities trade either in spot markets or derivatives markets. Spot markets are also referred to as “physical markets” or “cash markets” where buyers and sellers exchange physical commodities for immediate delivery (McDonald & Marsh, 1999). Derivatives markets involve forwards, futures and options. Forwards and futures are derivatives contracts that use the spot market as the underlying at some point in the future for a price agreed upon today. Only when the contracts expire would physical delivery of the commodity or other asset take place, and often traders will roll over or close out their contracts in order to avoid making or taking delivery altogether. Forwards and futures are generally the same, except that forwards are customizable and trade over-the-counter (OTC), whereas futures are standardized and traded on exchanges.

Equilibrium

Equilibrium price is the market price at which the quantity of goods supplied in the market by producers is equal to the quantity of goods demanded in the market by consumers. Equilibrium quantity, is the quantity demanded by consumers and the quantity supplied by producers at the equilibrium price. Ionu, Critina, Mihail, Anca, Simona, Elena & Mihaela (2021) ascertained that every production and consumption exert considerable environmental pressure on climate change, deterioration of national ecosystems and adverse health impacts. The above statement is not only limited to the natural mineral resources such as the crude oil, electricity, lime stones etc, but also agricultural goods that consumers need on daily basis. So, having a good knowledge of the equilibrium price will greatly reduce the fear of the unknown. Many different commodity agreements were adopted with sole aim to stabilize prices, mostly in developing countries (Naziru, Duru, Auwal & Maikud, 2023).

A system is said to be in equilibrium when there is no tendency for change. Market equilibrium is achieved when there is no deficiency supply (shortage) or excess supply (Foster, 2016). In an isolated market model, the equilibrium condition consists of only one equation, $Q_d = Q_s, \, or \; E = Q_d = Q_s = 0, \, where \; E$ stands for excess demand. When several interdependent commodities are simultaneously considered, equilibrium would require the absence of excess demand for each and every commodity included in the model, for if so, much as one commodity is faced with an excess demand, the price adjustment of that commodity will necessarily affect the quantities demanded and quantities supplied of the related commodities, thereby causing price changes all round. (Tamas & Balas, 2022) investigates the extent to which the structure of the interaction network between supplier and buyers affects equilibrium price heterogeneity. Consequently, the equilibrium condition of an n-commodity market model will involve n equations, one for each commodity, in the form

$$E_i = Q_{di} - Q_{si} = 0 \qquad (1, 2, 3, \ldots, n \quad) \tag{1}$$

Determinant of a Matrix

Determinants were originally introduced for solving system of linear equation. Although impractical in computations, they have important engineering applications in eigenvalue problems, differential equations, vector algebra and so on (Kreyszig, 2006). They can be introduced in several equivalent ways.

A determinant of order n is a scalar associated with an $n \times n \, (hence \, square) \, matrix$

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

And is defined for $n = 1 \, by \quad D = a_{11}$

And for $n \geq 2 \, by \quad D = a_{j1}C_{j1} + a_{j2}C_{j2} + \ldots + a_{jn}C_{jn} \qquad (j = 1,2,..., or \; n)$

or

$$D = a_{1k}C_{1k} + a_{2k}C_{2k} + \ldots + a_{nk}C_{nk} \qquad (k = 1,2,..., or \; n)$$

Here, $C_{jk} = (-1)^{j+k} M_{jk}$

And M_{jk} is a determinant of order n-1, namely, the determinant of the matrix of ‘A’ of obtained from A by omitting the row and column of the entry a_{jk}, that is, the jth row and the kth column.

Cramer's Rule

Cramer's rule is a rule which opens the way to the classical solution formula for linear systems as quotients of determinants (Kreyszig, 2006).

If a linear system of n equations in the same number of unknown $x_1, \ldots, x_n$

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_2 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

....

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b$$

Has a nonzero coefficient, determinant D=det A, the system has precisely one solution. This solution is given by the formulas

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad \frac{D_3}{D}, \ldots, x_n = \frac{D_n}{D}$$

where D_k is the determinant obtained from D by replacing in D the kth column by the column with the entries $b_1, \ldots, b_n$.

The Model

Here we are developing a model in which four commodities are related to one another. For simplicity, the demand and supply functions of all commodities assumed to be linear. In parametric terms, such a model can be written as

$$Q_{d1} - Q_{s1} = 0 \qquad (2)$$

Q_d = quantity demanded

Q_s = quantity supplied

$$Q_{d1} = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4 \qquad (3)$$

$$Q_{s1} = b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 \qquad (4)$$

$$Q_{d2} - Q_{s2} = 0 \qquad (5)$$

$$Q_{d2} = \alpha_0 + \alpha_1P_1 + \alpha_2P_2 + \alpha_3P_3 + \alpha_4P_4 \qquad (6)$$

$$Q_{s2} = \beta_0 + \beta_1P_1 + \beta_2P_2 + \beta_3P_3 + \beta_4P_4 \qquad (7)$$

$$Q_{d3} - Q_{s3} = 0 \tag{8}$$

$$Q_{d3} = f_0 + f_1P_1 + f_2P_2 + f_3P_3 + f_4P_4 \tag{9}$$

$$Q_{s3} = g_0 + g_1P_1 + g_2P_2 + g_3P_3 + g_4P_4 \tag{10}$$

$$Q_{d4} - Q_{s4} = 0 \tag{11}$$

$$Q_{d4} = \gamma_0 + \gamma_1P_1 + \gamma_2P_2 + \gamma_3P_3 + \gamma_4P_4 \tag{12}$$

$$Q_{s4} = t_0 + t_1P_1 + t_2P_2 + t_3P_3 + t_4P_4 \tag{13}$$

Where $a \; and \; b$ are coefficients that pertain to the demand and supply functions of the first commodity, $\alpha \; and \; \beta$ to the second commodity, $f \; and \; g$ for the third commodity and $\gamma \; and \; t$.

As a first step toward the solution of this model, we can resort to elimination of variables. By substituting equation 3 and 4 into equation 2 for the first commodity, equation 6 and 7 into equation 5 for the second commodity, equation 9 and 10 into equation 8 for the third commodity and 12 and 13 into 11 for the fourth commodity. The model is reduced to four equations in eight variables:

$$(a_0 - b_0) + (a_1 - b_1)P_1 + (a_2 - b_2)P_2 + (a_3 - b_3)P_3 + (a_4 - b_4)P_4 = 0 \tag{14}$$

$$(\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)P_1 + (\alpha_2 - \beta_2)P_2 + (\alpha_3 - \beta_3)P_3 + (\alpha_4 - \beta_4)P_4 = 0 \tag{15}$$

$$(f_0 - g_0) + (f_1 - g_1)P_1 + (f_2 - g_2)P_2 + (f_3 - g_3)P_3 + (f_4 - g_4) = 0 \tag{16}$$

$$(\gamma_0 - t_0) + (\gamma_1 - t_1)P_1 + (\gamma_2 - t_2)P_2 + (\gamma_3 - t_3)P_3 + (\gamma_4 - t_4) = 0 \tag{17}$$

These represent the four-commodity version of (1), after the demand and supply functions have been substituted into the two equilibrium conditions.

Although this is a model of only four equations, there is already 30 parameters. Algebraic manipulations will prove unwieldy unless some sort of shorthand is introduced. Let us therefore defined the shorthand symbols as follows

$$\omega_i = a_i - b_i, \quad \phi_i = \alpha_i - \beta_i, \quad \Omega_i = f_i - g_i, \quad \psi_i = \gamma_i - t_i \quad (i = 0, 1, 2, 3, 4) \tag{18}$$

$$\omega_0 = a_0 - b_0, \; \omega_1 = a_1 - b_1, \; \omega_2 = a_2 - b_2, \quad \omega_3 = a_3 - b_3 \;\; \omega_4 = a_4 - b_4 \tag{19}$$

$$\phi_0 = \alpha_0 - \beta_0, \; \phi_1 = \alpha_1 - \beta_1, \; \phi_2 = \alpha_2 - \beta_2, \; \phi_3 = \alpha_3 - \beta_3 \;\; \phi_4 = \alpha_4 - \beta_4 \tag{20}$$

$$\Omega_0 = f_0 - g_0, \Omega_1 = f_1 - g_1, \; \Omega_2 = f_2 - g_2, \; \Omega_3 = f_3 - g_3 \;\; \Omega_4 = f_4 - g_4 \tag{21}$$

$$\psi_0 = \gamma_0 - t_0, \; \psi_1 = \gamma_1 - t_1, \; \psi_2 = \gamma_2 - t_2, \; \psi_3 = \gamma_3 - t_3, \; \psi_4 = \gamma_4 - t_4 \tag{22}$$

Making b_i, β_i, g_i *and* $t_i \quad i=1, 2, 3, 4$ the subject of the formula in equation 19, 20, 21 and 22 and substituting into equation 14, 15, 16 and 17 will yield the following set of equations:

$$\omega_1 P_1 + \omega_2 P_2 + \omega_3 P_3 + \omega_4 P_4 = -\omega_0 \tag{23}$$

$$\phi_1 P_1 + \phi_2 P_2 + \phi_3 P_3 + \phi_4 P_4 = -\phi_0 \tag{24}$$

$$\Omega_1 P_1 + \Omega_2 P_2 + \Omega_3 P_3 + \Omega_4 P_4 = -\Omega_0 \tag{25}$$

$$\psi_1 P_1 + \psi_2 P_2 + \psi_3 P_3 + \psi_4 P_4 = -\psi_0 \tag{26}$$

Representing equation 23 to 26 in matrix form we have the following

$$\underset{A}{\begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \\ \phi_1 & \phi_2 & \phi_3 & \phi_4 \\ \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 \\ \psi_1 & \psi_2 & \psi_3 & \psi_4 \end{bmatrix}} \underset{X}{\begin{bmatrix} \omega \\ \phi \\ \Omega \\ \psi \end{bmatrix}} = \underset{B}{\begin{bmatrix} -\omega \\ -\phi \\ -\Omega \\ -\psi \end{bmatrix}} \tag{27}$$

The equilibrium price for the four commodities could be derived using the Cramer's rule as follows:

Let A represent the 4 by 4 matrix in equation 27 above and $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ represent a 4 by 4 matrices derived from equation 27 by replacing column one in equation 27 with the B column which will now represent Δ_1, and for Δ_2, we replace column 2 with the B column. Δ_3 will be replaced with the third column and for Δ_4 the fourth column will be replaced with B column.

$$|A| = \omega_1 \begin{vmatrix} \phi_2 & \phi_3 & \phi_4 \\ \Omega_2 & \Omega_3 & \Omega_4 \\ \psi_2 & \psi_3 & \psi_4 \end{vmatrix} - \omega_2 \begin{vmatrix} \phi_1 & \phi_3 & \phi_4 \\ \Omega_1 & \Omega_3 & \Omega_4 \\ \psi_1 & \psi_3 & \psi_4 \end{vmatrix} + \omega_3 \begin{vmatrix} \phi_1 & \phi_2 & \phi_4 \\ \Omega_1 & \Omega_2 & \Omega_4 \\ \psi_1 & \psi_2 & \psi_4 \end{vmatrix} - \omega_4 \begin{vmatrix} \phi_1 & \phi_2 & \phi_{33} \\ \Omega_1 & \Omega_2 & \Omega_3 \\ \psi_1 & \psi_2 & \psi_3 \end{vmatrix}$$

$$\begin{aligned} |A| = {} & \omega_1 [\phi_2 \Omega_3 \psi_4 + \phi_3 \Omega_4 \psi_2 + \phi_4 \Omega_2 \psi_3 - \phi_4 \Omega_3 \psi_2 - \phi_3 \Omega_2 \psi_4 - \phi_2 \Omega_4 \psi_3] \\ & - \omega_2 [\phi_1 \Omega_3 \psi_4 + \phi_3 \Omega_4 \psi_1 + \phi_4 \Omega_1 \psi_3 - \phi_4 \Omega_3 \psi_1 - \phi_3 \Omega_1 \psi_4 - \phi_1 \Omega_4 \psi_3] \\ & + \omega_3 [\phi_1 \Omega_2 \psi_4 + \phi_2 \Omega_4 \psi_1 + \phi_4 \Omega_1 \psi_2 - \phi_4 \Omega_2 \psi_1 - \phi_2 \Omega_1 \psi_4 - \phi_1 \Omega_4 \psi_2] \\ & - \omega_4 [\phi_1 \Omega_2 \psi_3 + \phi_2 \Omega_3 \psi_1 + \phi_3 \Omega_1 \psi_2 - \phi_3 \Omega_2 \psi_1 - \phi_2 \Omega_1 \psi_3 - \phi_1 \Omega_3 \psi_2] \end{aligned} \tag{28}$$

For Δ_1, we replace the first column with the B column

$$|\Delta_1| = \begin{vmatrix} -\omega_0 & \omega_2 & \omega_3 & \omega_4 \\ -\phi_0 & \phi_2 & \phi_3 & \phi_4 \\ -\Omega_0 & \Omega_2 & \Omega_3 & \Omega_4 \\ -\psi_0 & \psi_2 & \psi_3 & \psi_4 \end{vmatrix}$$

$$|\Delta_1| = -\omega_0 \begin{vmatrix} \phi_2 & \phi_3 & \phi_4 \\ \Omega_2 & \Omega_3 & \Omega_4 \\ \psi_2 & \psi_3 & \psi_4 \end{vmatrix} - \omega_2 \begin{vmatrix} -\phi_0 & \phi_3 & \phi_4 \\ -\Omega_0 & \Omega_3 & \Omega_4 \\ -\psi_0 & \psi_3 & \psi_4 \end{vmatrix} + \omega_3 \begin{vmatrix} -\phi_0 & \phi_2 & \phi_4 \\ -\Omega_0 & \Omega_2 & \Omega_4 \\ -\psi_0 & \psi_2 & \psi_4 \end{vmatrix} - \omega_4 \begin{vmatrix} -\phi_0 & \phi_2 & \phi_3 \\ -\Omega_0 & \Omega_2 & \Omega_3 \\ -\psi_0 & \psi_2 & \psi_3 \end{vmatrix}$$

$$\begin{aligned} |\Delta_1| = &-\omega_0\left[\phi_2\Omega_3\psi_4 + \phi_3\Omega_4\psi_2 + \phi_4\Omega_2\psi_3 - \phi_4\Omega_3\psi_2 - \phi_3\Omega_2\psi_4 - \phi_2\Omega_4\psi_3\right] \\ &-\omega_2\left[(-\phi_0)\Omega_3\psi_4 + \phi_3\Omega_4(-\psi_0) + \phi_4(-\Omega_0)\psi_3 - \phi_4\Omega_3(-\psi_0) - \phi_3(-\Omega_0)\psi_4 - (-\phi_0)\Omega_4\psi_3\right] \\ &+\omega_3\left[(-\phi_0)\Omega_2\psi_4 + \phi_2\Omega_4(-\psi_0) + \phi_4(-\Omega_0)\psi_2 - \phi_4\Omega_2(-\psi_0) - \phi_2(-\Omega_0)\psi_4 - (-\phi_0)\Omega_4\psi_2\right] \\ &-\omega_4\left[(-\phi_0)\Omega_2\psi_3 + \phi_2\Omega_3(-\psi_0) + \phi_3(-\Omega_0)\psi_2 - \phi_3\Omega_2(-\psi_0) - \phi_2(-\Omega_0)\psi_3 - (-\phi_0)\Omega_3\psi_2\right] \end{aligned} \quad (29)$$

In a similar way, we have

$$\begin{aligned} |\Delta_2| = &\ \omega_1\left[(-\phi_0)\Omega_3\psi_4 + \phi_3\Omega_4(-\psi_0) + \phi_4(-\Omega_0)\psi_3 - \phi_4\Omega_3(-\psi_0) - \phi_3(-\Omega_0)\psi_4 - (-\phi_0)\Omega_4\psi_3\right] \\ &+\omega_0\left[\phi_1\Omega_3\psi_4 + \phi_3\Omega_4\psi_1 + \phi_4\Omega_1\psi_3 - \phi_4\Omega_3\psi_1 - \phi_3\Omega_1\psi_4 - \phi_1\Omega_4\psi_3\right] \\ &+\omega_3\left[\phi_1(-\Omega_0)\psi_4 + (-\phi_0)\Omega_4\psi_1 + \phi_4\Omega_1(-\psi_0) - \phi_4(-\Omega_0)\psi_1 - (-\phi_0)\Omega_1\psi_4 - \phi_1\Omega_4(-\psi_0)\right] \\ &-\omega_4\left[\phi_1(-\Omega_0)\psi_3 + (-\phi_0)\Omega_3\psi_1 + \phi_3\Omega_1(-\psi_0) - \phi_3(-\Omega_0)\psi_1 - (-\phi_0)\Omega_1\psi_3 - \phi_1\Omega_3(-\psi_0)\right] \end{aligned} \quad (30)$$

$$\begin{aligned} |\Delta_3| = &\ \omega_1\left[\phi_2(-\Omega_0)\psi_4 + (-\phi_0)\Omega_4\psi_2 + \phi_4\Omega_2(-\psi_0) - \phi_4(-\Omega_0)\psi_2 - (-\phi_0)\Omega_2\psi_4 - \phi_2\Omega_4(-\psi_0)\right] \\ &-\omega_2\left[\phi_1(-\Omega_0)\psi_4 + (-\phi_0)\Omega_4\psi_1 + \phi_4\Omega_1(-\psi_0) - \phi_4(-\Omega_0)\psi_1 - (-\phi_0)\Omega_1\psi_4 - \phi_1\Omega_4(-\psi_0)\right] \\ &-\omega_0\left[\phi_1\Omega_2\psi_4 + \phi_2\Omega_4\psi_1 + \phi_4\Omega_1\psi_2 - \phi_4\Omega_2\psi_1 - \phi_2\Omega_1\psi_4 - \phi_1\Omega_4\psi_2\right] \\ &-\omega_4\left[\phi_1\Omega_2(-\psi_0) + \phi_2(-\Omega_0)\psi_1 + (-\phi_0)\Omega_1\psi_2 - (-\phi_0)\Omega_2\psi_1 - \phi_2\Omega_1(-\psi_0) - \phi_1(-\Omega_0)\psi_2\right] \end{aligned} \quad (31)$$

$$\begin{aligned} |\Delta_4| = &\ \omega_1\left[\phi_2\Omega_3(-\psi_0) + \phi_3(-\Omega_0)\psi_2 + (-\phi_0)\Omega_2\psi_3 - (-\phi_0)\Omega_3\psi_2 - \phi_3\Omega_2(-\psi_0) - \phi_2(-\Omega_0)\psi_3\right] \\ &-\omega_2\left[\phi_1\Omega_3(-\psi_0) + \phi_3(-\Omega_0)\psi_1 + (-\phi_0)\Omega_1\psi_3 - (-\phi_0)\Omega_3\psi_1 - \phi_3\Omega_1(-\psi_0) - \phi_1(-\Omega_0)\psi_3\right] \\ &+\omega_3\left[\phi_1\Omega_2(-\psi_0) + \phi_2(-\Omega_0)\psi_1 + (-\phi_0)\Omega_1\psi_2 - (-\phi_0)\Omega_2\psi_1 - \phi_2\Omega_1(-\psi_0) - \phi_1(-\Omega_0)\psi_2\right] \\ &+\omega_0\left[\phi_1\Omega_2\psi_3 + \phi_2\Omega_3\psi_1 + \phi_3\Omega_1\psi_2 - \phi_3\Omega_2\psi_1 - \phi_2\Omega_1\psi_3 - \phi_1\Omega_3\psi_2\right] \end{aligned} \quad (32)$$

By definition of the Cramer's rule, we have the following prices

$$P_1^* = \frac{|\Delta_1|}{|A|}, \quad P_2^* = \frac{|\Delta_2|}{|A|} \quad P_3^* = \frac{|\Delta_3|}{|A|} \text{ and } P_4^* = \frac{|\Delta_4|}{|A|}$$

Numerical Example.

The demand and supply function for four interdependent commodities in a competitive market is defined as follows:

Apples

$$\begin{aligned} Q_{d1} &= 100 - 2P_1 + P_2 - P_3 + 2P_4 \\ Q_{s1} &= 50 + 3P_1 - 2P_2 + P_3 - P_4 \end{aligned} \tag{33}$$

Grapes

$$\begin{aligned} Q_{d2} &= 200 - 3P_2 + P_1 - P_3 + 3P_4 \\ Q_{s2} &= 100 + 2P_2 - P_1 + 2P_3 - 2P_4 \end{aligned} \tag{34}$$

Coffee

$$\begin{aligned} Q_{d3} &= 150 - 4P_3 + 2P_1 - 2P_2 + P_4 \\ Q_{s3} &= 75 + 5P_3 - 3P_1 + P_2 + 2P_4 \end{aligned} \tag{35}$$

Banana

$$\begin{aligned} Q_{d4} &= 50 - 5P_4 + 3P_1 - P_2 - 2P_3 \\ Q_{s4} &= 20 + 6P_4 - 2P_1 + 3P_2 + P_4 \end{aligned} \tag{36}$$

Where Q_{di}, Q_{si} *and* P denote the quantity demanded, quantity supplied and price of goods i respectively. In determining the equilibrium price and quantity for this four-commodity model, we proceed as follows:

From equation (2), $Q_{d1} - Q_{s1} = 0 \Rightarrow (100 - 2P_1 + P_2 - P_3 + 2P_4) - (50 + 3P_1 - 2P_2 + P_3 - P_4) = 0$

$$\Rightarrow 100 - 2P_1 + P_2 - P_3 + 2P_4 - 50 - 3P_1 + 2P_2 - P_3 + P_4 = 0$$

$$\Rightarrow 50 - 5P_1 + 3P_2 - 2P_3 + 3P_4 = 0$$

From the above equation, we obtain the values for the variables as shown bellow

$$\omega_0 = 50, \ \omega_1 = -5, \ \omega_2 = 3, \ \omega_3 = -2, \quad \omega_4 = 3$$

In a similar way, we have from equation (5),

$$\phi_0 = 100, \ \phi_1 = 2, \ \phi_2 = -5, \ \phi_3 = -3, \quad \phi_4 = 5$$

and from equation (8) and (11), we have

$\Omega_0 = 75,\ \Omega_1 = 5,\ \Omega_2 = -3,\ \Omega_3 = -9,\quad \Omega_4 = -1$ and

$\psi_0 = 30,\ \psi_1 = 5,\ \psi_2 = -4,\ \psi_3 = -3,\quad \psi_4 = -11$ respectively.

Now substituting the above values into equation (28), (29), (30), (31) and (32), we have the equilibrium prices as follows:

$$P_1^* = \frac{38{,}935}{2{,}165},\quad P_2^* = \frac{44{,}665}{2{,}165}\quad P_3^* = \frac{24{,}715}{2{,}165},\quad p_4^* = \frac{620}{2{,}165}$$

To show that the above prices are the equilibrium prices for the four commodities, we substitute the above values into equation 33, 34, 35 and 36. The result for equation 33 is 74. That is, the quantity demanded and the quantity supplied is 74. For equation 34, we have 146. For equation 35 and 6 we have 99 and 59 respectively.

Discussion of Result

In reality, we all know that the higher the price of any commodity the lower the quantities demanded. From the result presented above, we discovered that the quantities of Banana supplied is 59. This low quantity compare to the rest commodities might be as result of high price of banana. The result also shows that the demand for Grape is higher than other goods. This may also be as result of the existing market price for Grape.

Conclusion

Though dummy data were used for this model, the result shows the reality of what is happening in any commodity market. One of the controlling forces in a commodity market is the price. Consumers always try to get maximum satisfaction with their meagre resources. So, when the supply of goods with close substitute tends to increase or decrease in the market, it is as result of the existing market price. Because of the poor economic situation in many developing countries, especially in Africa, people no longer buy food because of with high quality rather they go for quantity. So, producers should produce goods that can meet the demand of the average man which constitute the greater population in developing countries. And of course, that should be one of the reasons why a good knowledge of the equilibrium price is so important in any commodity market.

In our present day, isolated-market wherein the Q_d *and* Q_s of a commodity are functions of the price of that commodity alone hardly exist because of the existence of many substitute and complementary goods, hence the need for a model that will handle the situation of goods that have substitutes. As earlier said, there will be an excess demand in an isolated-market model. But when several interdependent commodities are considered simultaneously, equilibrium will require the absence of excess demand for each and every commodity included in the model.

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