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OPTIMIZATION OF MULTIPHASE QUEUING MODELS FOR ANTENATAL CARE UNITS IN NIGERIAN PUBLIC HOSPITALS

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Abstract

Efficient healthcare delivery is crucial for improving population health outcomes, especially in maternal and child health services in low- and middle-income countries. One of the persistent operational challenges in healthcare institutions is managing patient flow to minimize waiting times while ensuring optimal use of available resources. This study investigates the use of multiphase queuing models to optimize patient flow at the antenatal clinic of the Federal Medical Centre (FMC) Umuahia, Nigeria. The study adopts a (M/Ek/1) :(FCFS/∞/∞) queuing model and explores three other queuing models to check system efficiency. Primary data were collected through direct observations, structured interviews with staff, and patient surveys. The TORA and R programming software were used to analyse system performance, test of hypothesis and validity of model assumptions. The findings reveal significant differences in system performance for the different queuing models, with hybrid(single-cum-multi-server) models demonstrating superior outcomes in terms of reduced waiting time, and balanced utilization. This research contributes to healthcare operations management by providing evidence-based strategies for enhancing service delivery and patient satisfaction in resource-constrained maternal healthcare settings.

Keywords: Queuing theory, antenatal care, service optimization, multiphase model, healthcare efficiency, maternal health, server utilization, patient waiting time.

Introduction

One of the most critical components of a well-functioning healthcare system is its ability to provide timely and efficient services to its clients. In developing countries such as Nigeria, the healthcare infrastructure is often overstretched due to increasing population growth, limited health personnel, and insufficient medical facilities. As a result, healthcare institutions particularly those offering essential services like antenatal care frequently experience long patient queues and excessive waiting times. These delays do not only reduce patient satisfaction, but may also have adverse effects on maternal and fetal health outcomes. Poor queue management in developing countries leads to high patient attrition and dissatisfaction.

Antenatal care (ANC) is a vital aspect of maternal health services aimed at monitoring and ensuring the well-being of both mother and baby throughout the course of pregnancy. Given the clinical importance of ANC, optimizing the delivery of such services is a public health priority. In Nigerian public hospitals, particularly tertiary centres like the Federal Medical Centre (FMC) Umuahia, ANC units are often characterized by inefficient patient flow due to high patient volume, health record issues, poorly managed scheduling systems, and inconsistent staffing across service points.

This study is motivated by the urgent need to address these operational inefficiencies by applying quantitative and data-driven methods. Queuing theory as a branch of operations research, offers a robust analytical tool for modelling patient flow and identifying areas for process improvement. By examining the queuing characteristics of the ANC unit at FMC Umuahia, this study aims to provide evidence-based insights into how queuing configurations specifically multiphase queuing systems can be optimized to enhance patient throughput, reduce waiting time, and improve resource utilization. The ultimate goal of this study, is to derive actionable recommendations for hospital administrators and policymakers aimed at improving service delivery in maternal healthcare settings.

The application of queuing theory in healthcare systems has been widely explored across various domains including outpatient clinics, emergency departments, pharmacies, and operating theatres. Queuing theory provides a quantitative foundation for modelling patient flow, scheduling appointments, optimizing resource allocation, and minimizing patient waiting times. Fomundam and Herrmann (2007), offered a comprehensive survey of queuing theory applications in healthcare, emphasizing the importance of queue models in patient scheduling, service design, and capacity planning. Their work particularly emphasized the utility of queuing models in improving performance across different scales of healthcare from individual departments to regional healthcare systems. Adan and Resing (2002), focused on analytical solution techniques for complex queuing systems, highlighting the use of Markov processes in solving healthcare related queues involving multiple service stages. Their analysis demonstrated how different queue disciplines and service distributions affect system stability and performance. Adeleke *et al.* (2005), discussed the role of queue management in enhancing service delivery within Nigerian healthcare systems. They emphasized that uncertainty in service time and arrival patterns leads to congestion and reduced patient satisfaction, recommending the use of stochastic models to better understand patient behaviour. Ekpenyong and Udoh (2011) developed and extended queuing models to account for multiple service phases and multi-server settings. Their modified models improved on traditional single-server frameworks and were validated using real hospital data.

While significant progress has been made in applying queuing models to healthcare, there remains a notable gap in localized studies focusing on maternal health services in Nigeria. Obamiro (2011), applied queuing theory in the analysis of antenatal clinic operations in Nigeria, using tools like the TORA optimization software. His findings highlighted inefficiencies in patient flow and proposed multiphase queuing models as effective alternatives for reducing wait times and balancing server loads. Recent studies have highlighted persistent delays in antenatal care (ANC) services across Nigerian healthcare facilities. Okonofua *et al*. (2023) conducted a multi-site study revealing significant waiting times particularly in outpatient departments, with an average total time of 237.6 minutes spent by women receiving ANC services. This underscores the need for operational improvements to enhance patient flow and reduce delays. Together, these studies establish a solid

foundation for this research. It builds upon the existing literature by exploring multiphase queuing models in ANC of a Nigerian public hospital, and evaluating queuing performance specifically within an antenatal clinic setting using both empirical data, and statistical modelling.

Methodology

The multiphase nature of antenatal care, which typically involves sequential stages such as registration, vital sign checks, counselling, and doctor consultation, makes it suitable for analysis through a multiphase queuing framework. This study models the antenatal unit using the (M/Ek/1) :(FCFS/∞/∞) notation, capturing the stochastic characteristics of patient arrivals and service times. Additionally, the study explores various server configurations to determine the most efficient system design. This study adopts a descriptive case study approach, focusing on the antenatal clinic of the Federal Medical Centre (FMC) Umuahia, a tertiary healthcare institution in south-eastern Nigeria. The methodology is divided into four key components: data collection, model formulation, performance evaluation, and statistical analysis.

Data Collection

Data was collected over a five-day period through direct observation, structured interviews, and patient surveys:

Observation: Arrival and service times were manually recorded for each patient using timing sheets and stopwatches. The observed service points were registration (folder retrieval), blood pressure check, height and weight measurement, and medical consultation. Inter-arrival and service times were recorded for each phase.

Structured Interviews: Healthcare providers, administrative staff, and ANC unit managers were interviewed to provide contextual information on staffing levels, process flow, and challenges in service delivery.

Patient Surveys: A structured questionnaire was administered to antenatal patients to evaluate their perception of service quality, acceptable waiting time thresholds, satisfaction levels, and suggestions for system improvement. Data on demographics and frequency of clinic visits were also collected.

Queuing Model Formulation

The antenatal care service system was modelled as a multiphase queuing system using Kendell's notation (M/Ek/1): (FCFS/∞/∞). Where M represents Markovian arrival time, E_k represents Erlang service time, 1 indicating single server at each phase, (∞, ∞) measures the capacity of the system in terms of unlimited number of patients at the service stations, and infinite population source. Key assumptions of the model include:

- Patient arrivals follow a Poisson distribution.
- Service times follow an Erlang distribution with shape parameter k.
- Service is provided on a first-come-first-served basis.
- The system has unlimited queue capacity and population size.

The model configuration is illustrated in the figure below and labelled as model I

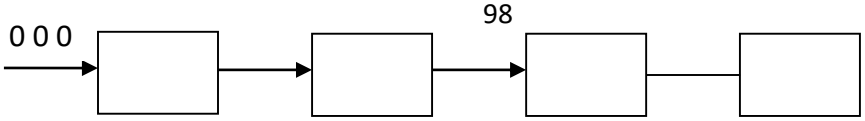

Figure1: Single server-multiphase queuing model (Erlang model I)

Other queuing models experimented for the system are illustrated in figures 2-4 below.

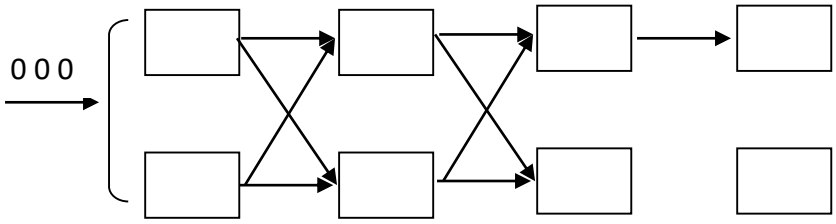

Figure 2: Multi sever multiphase queuing model (Erlang Model II)

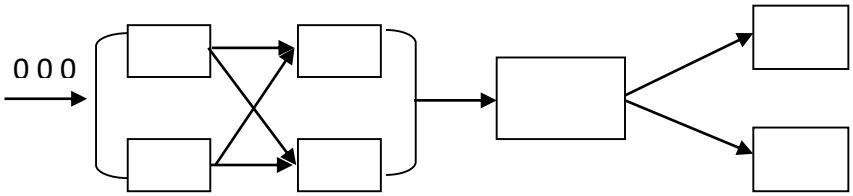

Figure 3: Single-cum-multi server multiphase queuing Model (Erlang Model III)

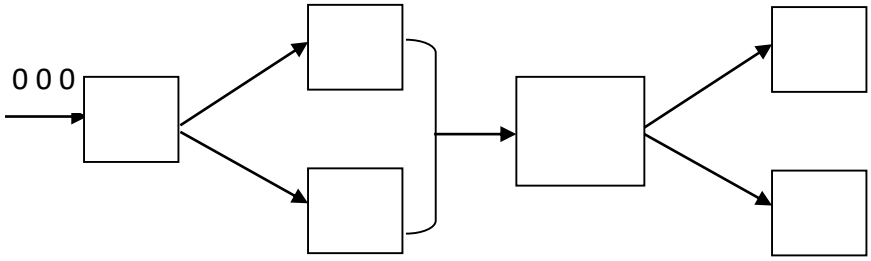

Figure 4: Single-cum-Multi server Queuing Model (Erlang Model IV)

Performance Evaluation Metrics

For each model, the following queuing performance metrics were computed using TORA software:
Lq: Average number of patients in queue, **Ls**: Average number of patients in the system,
Wq: Average waiting time in queue, **Ws**: Average time a patient spends in the system
and ρ (rho): Server utilization rate.

Statistical Validation and Analysis

To verify the assumptions of the queuing models, Chi-square tests were conducted to confirm Poisson arrival distribution and Erlang service distribution. One-way ANOVA tests were conducted to compare waiting times and server utilization across the four models. Tukey's Honest Significant Difference (HSD) tests were performed to determine pairwise differences between models. All statistical analyses were conducted using R programming language.

Data Analysis and Results

This section presents the outcomes of the queuing analysis performed on the data collected from the FMC Umuahia antenatal clinic. The results are organized based on the objectives: testing queuing assumptions, evaluating performance metrics for each queuing model, and comparing models using statistical tests.

Validation of Queuing Assumptions

Chi-square goodness-of-fit tests were conducted to validate the assumption that patient arrivals follow a Poisson process and that service times conform to an Erlang distribution: The chi-square test showed that the inter-arrival times did not deviate significantly from a Poisson distribution ($\chi^2 = 6.21$, d f = 8, p = 0.62). The service times were consistent with an Erlang-k distribution, confirmed through a chi-square test ($\chi^2 = 5.87$, d f = 6, p = 0.55). These results affirm the appropriateness of the (M/Ek/1) :(FCFS/∞/∞) model for the queuing analysis.

3.2 Performance Evaluation of Queuing Models

Each of the four experimental models was simulated and analysed using TORA to determine key performance metrics. The results are summarized in the table below:

Metric	**Model I**	**Model II**	**Model III**	**Model IV**
Wq (min)	8.8	0.2	1.9	3.8
Ws (min)	12.2	2.9	4.9	6.1
Lq (patients)	4	0	1	2
ρ(utilization)	0.9	0.5	0.6	0.7

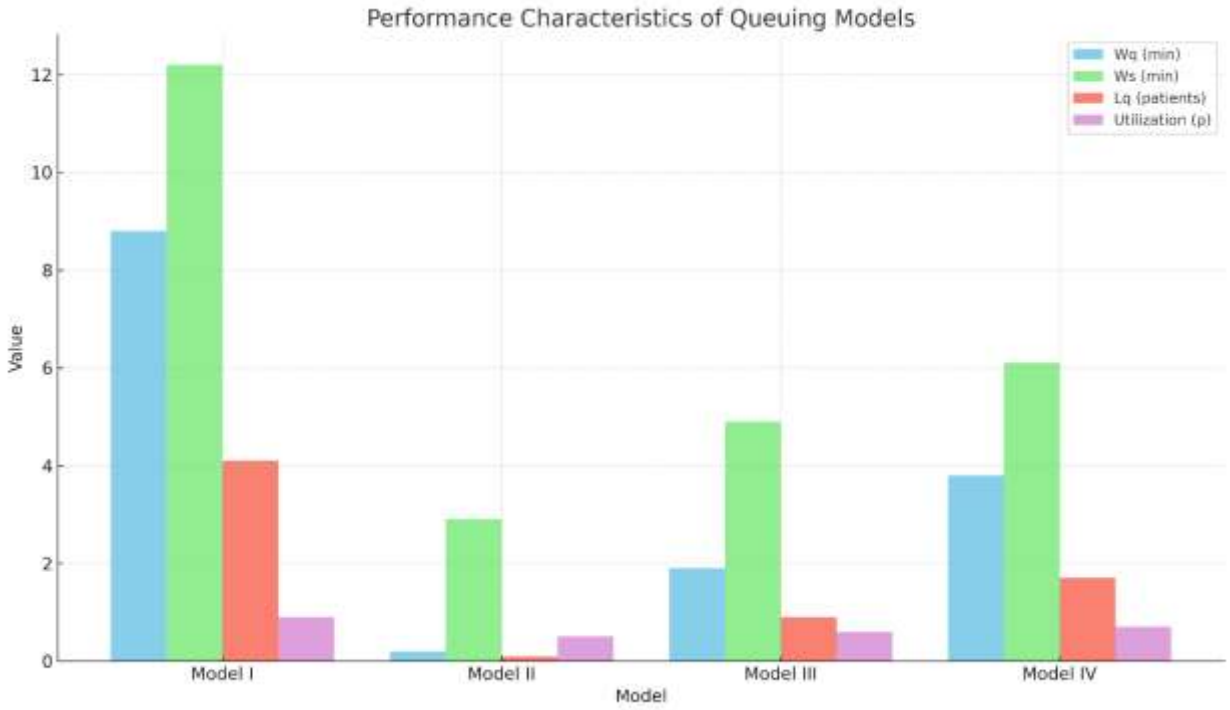

Fig 1: Comparison of queuing models I-IV

These results indicate that model II is the most efficient in terms of performance parameters although it also exhibited the lowest server utilization. However, model III offers the best trade-off, balancing cost efficiency and performance in a resource constrained environment. Therefore, model III is the optimal model for the system.

Statistical Analysis of Model Differences

A one-way ANOVA was conducted to test for significant differences in waiting time (Wq) and server utilization (ρ) among the four queuing models waiting Time: $F(3, 16) = 13.66$, $p = 0.0043$, utilization: $F(3, 16) = 325.70$, $p < 0.00$. The ANOVA results reveal statistically significant differences across models for both key performance indicators. Post-hoc analysis using Tukey's HSD test showed the following: Waiting times in Model II were significantly lower than those in Models I and IV ($p < 0.01$). Model I had the highest server utilization, significantly different from Models II and III ($p < 0.001$).

Summary of Findings

The analysis demonstrates that multi-server configurations significantly reduce patient waiting time and system congestion. There is a trade-off between efficiency (low waiting time) and utilization (high server usage). Model III provides a balance between performance and resource use, making it a practical choice for implementation. These findings confirm the effectiveness of multiphase queuing optimization in improving healthcare delivery in antenatal units.

Conclusions and Recommendations

This study investigated the application of multiphase queuing models to optimize patient flow and service efficiency at the antenatal clinic of the Federal Medical Centre (FMC) Umuahia, Nigeria. Using a (M/Ek/1) :(FCFS/∞/∞) model and three experimental queue configurations, the research examined the impact of different server allocations on key performance indicators such as waiting time, queue length, and server utilization. The analysis, supported by statistical validation and empirical data, revealed that server configurations significantly influence patient wait times and system performance.

The results showed that Model II with multi-servers at each service phase yielded the lowest waiting times and queue lengths, suggesting that resource distribution across phases reduces bottlenecks and enhances throughput. However, this configuration also resulted in lower server utilization, indicating potential underuse of resources in a cost-sensitive setting. Model III, which applied multiple servers strategically at high volume phases, presented a balanced performance profile, making it a feasible compromise in constrained healthcare environments. Howbeit, this work is limited to a five- day study since appointment schedules for pregnant women are tied to the clinic days of the patients' medical team. The implication of this is that until the patient is delivered of the baby the same patients triage at the clinic on particular days of the week. Overall, the study confirms the value of queuing theory as a tool for evaluating and optimizing maternal healthcare services. It demonstrates that informed modifications to server configurations can substantially improve service delivery without necessitating wholesale infrastructural changes.

Given resource limitations in most Nigerian public hospitals, a phased implementation of Model III is recommended. By deploying additional staff at critical bottleneck phases (such as registration and consultation), significant reductions in waiting time can be achieved without compromising cost efficiency. Hospitals should incorporate routine data collection on patient flow, service time, and queue lengths. These data can feed into updated queuing models, allowing administrators to make evidence-based adjustments over time. Health administrators should use queuing metrics such as server utilization (ρ) and average waiting time (Wq) to plan staff schedules and optimize resource allocation. Over-reliance on single-server configurations should be avoided in high-demand periods. The insights from this study should be considered by policymakers and hospital administrators when designing maternal health programs and funding allocations. Investment in operational efficiency improvements can yield high returns in service quality.

Future research can extend this work by integrating discrete event simulation models. Comparative studies across multiple healthcare institutions would also provide a broader foundation for generalized policy recommendations.

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